

The string prediction models as application to financial forex market

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In this paper we apply a new approach of the string theory to the real financial market. The strings are defined here by the boundary conditions, characteristic length, real values and the method of redistribution of information. The map represents the detrending and data standardization procedure. We used 1-end-point, 2-end-point open string and partially compactified strings that satisfy the Dirichlet and Neumann boundary conditions. We established two different models to predict the behavior of financial forex market. The first model is based on the correlation function as invariant and the second one is an application based on the deviations from the closed string/pattern form. We found the difference between these two approaches. The first model cannot predict the behavior of the forex market with good efficiency in comparison with the second one which is, in addition, able to make relevant profit per year.

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I. INTRODUCTION

In this paper we apply the string model and approaches described in [1] to real finance forex market. This is an extension of the previous work [1] into the real finance market. We derive two models for predictions of EUR/USD prices on the forex market. This is the first attempt for real application of the string theory in the field of finance, and not only in high energy physics, where it is established very well. Firstly we described briefly some connections between these different fields of research.

We would like to transfer modern physical ideas into the neighboring field called econophysics. The physical statistical viewpoint has proved to be fruitful, namely in the description of systems where many-body effects dominate. However, standard, accepted by physicists, bottom-up approaches are cumbersome or outright impossible to follow the behavior of the complex economic systems, where autonomous models encounter intrinsic variability.

Modern digital economy is founded on data. Our primary motivation comes from the actual physical concepts [2, 3]; however, our realization differs from the original attempts in various significant details. Similarly as with most science problems, the representation of data is the key to efficient and effective solutions. The string theory development over the past 25 years has achieved a high degree of popularity among physicists [4].

The underlying link between our approach and the string theory may be seen in the switching from a local to a non-local form of data description. This line passes from the single price to the multivalued collection, especially the string of prices from the temporal neighborhood, which we term here the string map. It is the relationship between more intuitive geometric methods and financial data. Here we work on the concept that is based on projection data into higher dimensional vectors in the sense of the works [5, 6].

The present work exploits time-series which can build the family of the string-motivated models of boundary-respecting maps. The purpose of the present data-driven study is to develop statistical techniques for the analysis of these objects and moreover for the utilization of such string models onto the forex market. Both of the string prediction models in this paper are built on the physical principle.

II. DEFINITION OF THE STRINGS

By applying standard methodologies of detrending one may suggest to convert original series of the quotations of the mean currency exchange rate $p(\tau)$ onto a series of returns defined by

$$\frac{p(\tau + h) - p(\tau)}{p(\tau + h)}, \quad (1)$$

where h denotes a tick lag between currency quotes $p(\tau)$ and $p(\tau + h)$, τ is the index of the quote. The mean $p(\tau) = (p_{\text{ask}}(\tau) + p_{\text{bid}}(\tau))/2$ is calculated from $p_{\text{ask}}(\tau)$ and $p_{\text{bid}}(\tau)$.

In the spirit of the string theory it would be better to start with the 1-end-point open string map

$$P^{(1)}(\tau, h) = \frac{p(\tau + h) - p(\tau)}{p(\tau + h)}, \quad h \in (0, l_s) \quad (2)$$

where the superscript (1) refers to the number of endpoints.

Later, we may use the notation $P\{p\}$ which emphasizes functional dependence upon the currency exchange rate $\{p\}$. It should also be noted that the use of P highlights the canonical formal correspondence between the *rate of return* and the internal *string momentum*.

Here the tick variable h may be interpreted as a variable which extends along the extra dimension limited by the string size l_s . A natural consequence of the transform, Eq.(2), is the fulfillment of the boundary condition

$$P^{(1)}(\tau, 0) = 0, \quad (3)$$

which holds for any tick coordinate τ . Later on, we want to highlight effects of the rare events. For this purpose, we introduce a power-law Q-deformed model

$$P_q^{(1)}(\tau, h) = \left(1 - \left[\frac{p(\tau)}{p(\tau + h)}\right]^Q\right), \quad Q > 0. \quad (4)$$

The 1-end-point string has defined the origin, but it reflects the linear trend in $p(\cdot)$ at the scale l_s . Therefore, the 1-end-point string map $P_q^{(1)}(\cdot)$ may be understood as a Q-deformed generalization of the *currency returns*.

Clearly, the situation with a long-term trend is partially corrected by fixing $P_q^{(2)}(\tau, h)$ at $h = l_s$. The open string with two end points is introduced via the nonlinear map which combines information about trends of p at two sequential segments

$$P_q^{(2)}(\tau, h) = \left(1 - \left[\frac{p(\tau)}{p(\tau + h)}\right]^Q\right) \left(1 - \left[\frac{p(\tau + h)}{p(\tau + l_s)}\right]^Q\right), \quad h \in]0, l_s[. \quad (5)$$

The map is suggested to include boundary conditions of *Dirichlet type*

$$P_q^{(2)}(\tau, 0) = P_q(\tau, l_s) = 0, \quad \text{at all ticks } \tau. \quad (6)$$

In particular, the sign of $P_q^{(2)}(\tau, h)$ comprises information about the behavior differences of $p(\cdot)$ at three quotes $(\tau, \tau + h, \tau + l_s)$. The $P_q^{(2)}(\tau, h) < 0$ occurs for trends of the different sign, whereas $P_q^{(2)}(\tau, h) > 0$ indicates the match of the signs.

Now we define partially compactified strings. In the frame of the string theory, the compactification attempts to ensure compatibility of the universe based on the four observable dimensions with twenty-six dimensions found in the theoretical model systems. From the standpoint of the problems considered here, the compactification may be viewed as an act of the information reduction of the original signal data, which makes the transformed signal periodic. Of course, it is not very favorable to close strings by the complete periodization of real input signals. Partial closure would be more interesting. This uses pre-mapping

$$\tilde{p}(\tau) = \frac{1}{N_m} \sum_{m=0}^{N_m-1} p(\tau + l_s m), \quad (7)$$

where the input of any open string (see e.g. Eq.(2), Eq.(5)) is made up partially compact.

Thus, data from the interval $[\tau, \tau + l_s(N_m - 1)]$ are being pressed to occupy "little space" $h \in]0, l_s[$. We see that as N_m increases, deviations of \tilde{p} from the periodic signal become less pronounced. For example, one might consider the construction of the $(\tilde{D} + 1)$ -brane

$$f_q \left(\frac{p(\tau + h_0) - p(\tau)}{p(\tau + h_0)} \right) \prod_{j=1}^{\tilde{D}} f_q \left(\frac{\tilde{p}_j^{(\pm)}(\tau + h_j) - \tilde{p}_j^{(\pm)}(\tau)}{\tilde{p}_j^{(\pm)}(\tau + h_j)} \right), \quad (8)$$

by means of the function

$$f_q(x) = \text{sign}(x) |x|^Q, \quad Q > 0 \quad (9)$$

maintained by combining $(\tilde{D} + 1)$ 1-end-point strings, where partial compactification in \tilde{D} extra dimensions is assumed. Of course, the construction introduces auxiliary variables $\tilde{p}_j^{(\pm)}(\tau) = \sum_{m=0}^{N_m, j-1} p(\tau \pm m l_{s, j})$. The corresponding statistical characteristics of all the strings and brane described above were displayed in detail in [1]. The prediction models were tested on the tick by tick one year data of EUR/USD major currency pair from ICAP market maker. More precisely, we selected period from October 2009 to September 2010.

III. CORRELATION FUNCTION AS INVARIANT

The meaning of invariant is that something does not change under transformation, e.g. such as some equations from one reference frame to another. We want to extend this idea also on the finance market and find there some invariants in the finance data and utilize this as a prediction for the next prices. Unfortunately this model is able to define only one step prediction, see definition below.

We suppose the invariant in the form of the correlation function

$$C_{(t, l_0)} = \sum_{h=l_0}^{h=l} w_h \left(1 - \frac{p_{t-h}}{p_{t-1-h}}\right) \left(1 - \frac{p_{t-1-h}}{p_{t-2-h}}\right), \quad (10)$$

with

$$w_h = \frac{e^{-h/\lambda}}{\sum_{h'=0}^l e^{-h'/\lambda}}, \quad (11)$$

including dependence on the time scale parameters l, l_0 and λ . The relative weights satisfy automatically $\sum_{h=0}^l w_h = 1$. From the condition of the invariance at the step $(t+1)$

$$C_{(t,0)} = C_{(t+1,0)} \simeq \hat{C}_{(t+1,0)} \quad (12)$$

we finally obtain the prediction

$$\hat{p}_{t+1} = p_t \left(1 + \frac{C_{(t+1,1)} - C_{(t,0)}}{w_0 \left(1 - \frac{p_t}{p_{t-1}} \right)} \right), \quad (13)$$

valid for $p_t \neq p_{t-1}$. These are general definitions for the correlation invariants.

A. Prediction model based on the string invariants

Now we want to take the above-mentioned ideas onto the string maps of finance data. We would like to utilize the power of the nonlinear string maps of finance data and establish some prediction models to predict the behavior of market similarly as in the works [7–9]. We suggest the method where one string is continuously deformed into the other. We analyze 1-end-point and 2-end-point mixed string models. The family of invariants is written using the parametrization

$$\begin{aligned} C(\tau, \Lambda) &= (1 - \eta_1)(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \\ &\times \left(1 - \left[\frac{p(\tau)}{p(\tau+h)} \right]^Q \right) \left(1 - \left[\frac{p(\tau+h)}{p(\tau+l_s)} \right]^Q \right) \\ &+ \eta_1(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \left(1 - \left[\frac{p(\tau)}{p(\tau+h)} \right]^Q \right) \\ &+ \eta_2 \sum_{h=0}^{\Lambda} W(h) \left(1 - \left[\frac{p(\tau+h)}{p(\tau+l_s)} \right]^Q \right), \end{aligned} \quad (14)$$

$$\quad (15)$$

where $\eta_1 \in (-1, 1)$, $\eta_2 \in (-1, 1)$ are variables (variables which we may be called homotopy parameters), Q is a real valued parameter, and the weight $W(h)$ is chosen in the bimodal single parameter form

$$W(h) = \begin{cases} 1 - W_0, & h \leq l_s/2, \\ W_0, & h > l_s/2. \end{cases} \quad (16)$$

We plan to express $p(\tau + l_s)$ in terms of the auxilliary variables

$$A_1(\Lambda) = (1 - \eta_1)(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \left(1 - \left[\frac{p(\tau)}{p(\tau+h)} \right]^Q \right), \quad (17)$$

$$A_2(\Lambda) = -(1 - \eta_1)(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \left(1 - \left[\frac{p(\tau)}{p(\tau+h)} \right]^Q \right) p^Q(\tau+h), \quad (18)$$

$$A_3(\Lambda) = \eta_1(1 - \eta_2) \sum_{h=0}^{\Lambda} W(h) \left(1 - \left[\frac{p(\tau)}{p(\tau+h)} \right]^Q \right), \quad (19)$$

$$A_4(\Lambda) = \eta_2 \sum_{h=0}^{\Lambda} W(h), \quad (20)$$

$$A_5(\Lambda) = -\eta_2 \sum_{h=0}^{\Lambda} W(h) p^Q(\tau+h). \quad (21)$$

Thus the expected prediction form reads

$$\hat{p}(\tau_0 + l_{\text{pr}}) = \left[\frac{A_2(\Lambda) + A_5(\Lambda)}{C(\tau_0 - l_s, \Lambda) - A_1(\Lambda) - A_3(\Lambda) - A_4(\Lambda)} \right]^{1/Q}, \quad (22)$$

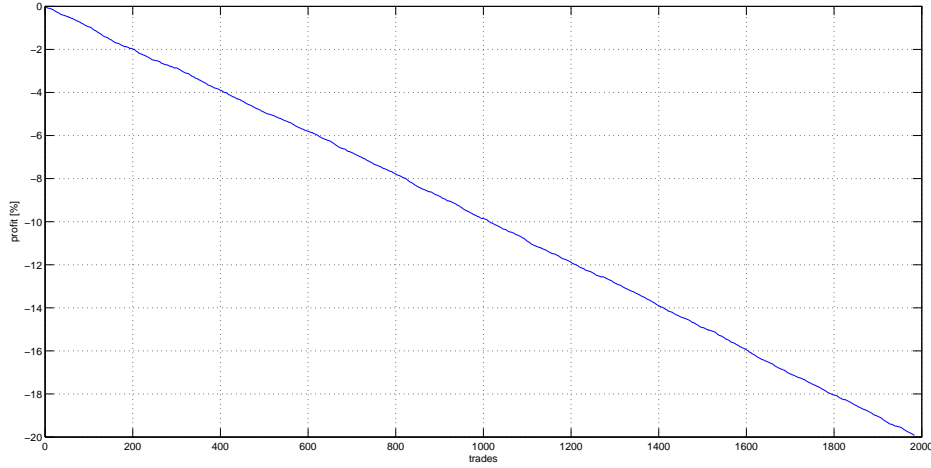


FIG. 1. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

where we use the notation $\tau = \tau_0 + l_{\text{pr}} - l_s$. The derivation is based on the invariance

$$C(\tau, l_s - l_{\text{pr}}) = C(\tau - l_{\text{pr}}, l_s - l_{\text{pr}}), \quad \Lambda = l_s - l_{\text{pr}}, \quad (23)$$

where l_{pr} denotes the prediction scale.

The model was tested for various sets of parameters l_s , l_{pr} , η_1 , η_2 , Q and the new parameter ϵ which is defined as

$$\epsilon = |C(\tau, l_s - l_{\text{pr}}) - C(\tau - l_{\text{pr}}, l_s - l_{\text{pr}})| \quad (24)$$

and describes the level of invariance in real data. The best prediction (the best means that the model has the best ability to guess the right price) is obtained by using the following values of parameters

$$\begin{aligned} l_s &= 900, \\ l_{\text{pr}} &= 1, \\ \eta_1 &= 0, \\ \eta_2 &= 0, \\ Q &= 6, \\ \epsilon &= 10^{-10}. \end{aligned} \quad (25)$$

The graphical descriptions of the prediction behavior of the model with and without transaction costs on the EUR/USD currency rate of forex market are described in Figs 1-4. During one year period the model lost around 20% of initial money. It executed 1983 trades (Fig 1) where only 10 were suggested by the model (and earned money) and the rest of them were random (which can be clearly seen in Figs 3,4). The problem of this model is its prediction length (the parameter l_{pr}), in this case it is one tick ahead. The price was predicted correctly in 48.57% of all cases (16201 in one year) and from these 48.57% or numerally 7869 cases only 0.13% or numerally 10 were suitable for trading. This small percentage is caused by the fact that the price does not change too often one tick ahead. One could try to raise the prediction length to find more suitable cases for trading. This is only partly successful because the rising parameter l_{pr} induces a loss of the prediction strength of the model. For example when $l_{\text{pr}} = 2$ (two ticks ahead) prediction strength decreases from around 50% to 15%.

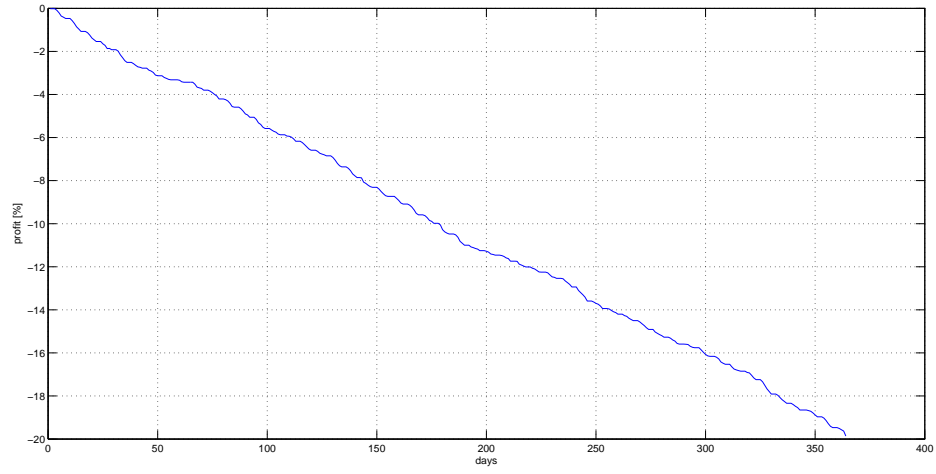


FIG. 2. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on days for one year period.

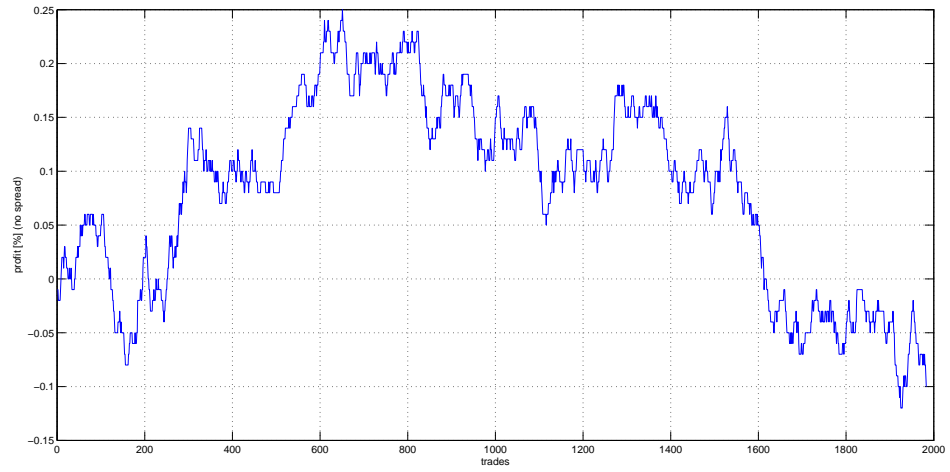


FIG. 3. The profit of the model on the EUR/USD currency rate without transaction costs included dependence on trades for one year period.

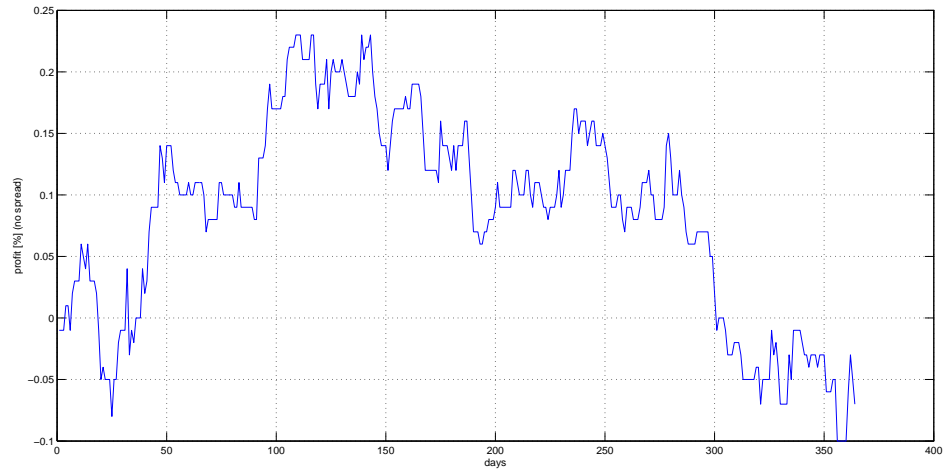


FIG. 4. The profit of the model on the EUR/USD currency rate without transaction costs included dependence on days for one year period.

IV. PREDICTION MODEL BASED ON THE DEVIATIONS FROM THE CLOSED STRING/PATTERN FORM

For the next trading strategy we want to define some real values of the string sequences. Therefore we define the momentum which acquired values from the interval $(0, 1)$. To study the deviations from the benchmark string sequence we define momentum with some regular function $F_{CS}(h/l_s)$ as

$$M_{(l_s; Q)}(\tau + l_s) = \left[\frac{1}{l_s + 1} \sum_{h=0}^{l_s} \left| p_{\text{stand}}(h, \tau; l_s) - F_{CS} \left(\frac{h}{l_s} \right) \right|^Q \right]^{\frac{1}{Q}}, \quad (26)$$

where

$$p_{\text{stand}}(\tau + h; l_s) = \frac{p(\tau + h) - p_{\min}(\tau; l_s)}{p_{\max}(\tau; l_s) - p_{\min}(\tau; l_s)}, \quad p_{\text{stand}} \in (0, 1),$$

and

$$p_{\max}(\tau; l_s) = \max_{h \in \{0, 1, 2, \dots, l_s\}} p(\tau + h), \quad p_{\min}(\tau; l_s) = \min_{h \in \{0, 1, 2, \dots, l_s\}} p(\tau + h).$$

The regular function from the definition above for closed string has to fulfill the condition $F_{CS}(0) = F_{CS}(1) = 1$, where $F_{CS} \in (0, 1)$.

A. Elementary trading strategy based on the probability density function of M

Effort is to take advantage whenever the market conditions are favorable.

We can image the set of instructions:

- The series of $(l_s + 1)$ price ticks $[p(\tau), p(\tau + 1), \dots, p(\tau + l_s)]$ is transformed into single representative real value $M(\tau + l_s)$;
- nearly stationary series of $M(\tau + l_s)$ yields statistics which can be split into:
 - branch where M is linked with future **uptrend/downtrend**;
 - branch where M is linked with future **profit/loss** taking into account **transaction costs**;
- accumulation of $\text{pdf}(M_{\text{long}}^{+-})$ (profit+ / loss-); $\text{pdf}(M_{\text{short}}^{+-})$ (profit+ / loss-);

Arbitrage opportunity - taking advantage of the occurrence of difference in distribution

Opportunity is measured by *Kullback-Liebler* divergence

$$D_{\text{KL}} = \sum_{j(\text{bins})} \text{pdf}(M^+(j)) \log \left(\frac{\text{pdf}(M^+(j))}{\text{pdf}(M^-(j))} \right)$$

- larger D_{KL} means better opportunities ($D_{\text{KL}} > D_{\text{threshold}}$);
- Statistical significance: the smaller the statistics accumulated into bins $\text{pdf}(M^+(j))$, $\text{pdf}(M^-(j))$, the higher is the risk (M from selected range should be widespread).

Again as in the previous section the model was tested for various sets of free parameters l_s , h , Q and the parameters used in the function F_{CS} . This model can make “more-tick” predictions (in tests it varies from 100 to 5000 ticks). Therefore it is much more successful than the previous model. It is able to make final profit of around 160% but this huge profit precedes a fall down of 140% of the initial state. It is important to emphasize that all profits mentioned here and below are achieved by using leverage (borrowing money) from 1 to 10. The reason for leverage is the fact that the model could simultaneously open up to 10 positions (one position means one trade i.e. one pair of buy-sell transactions). If one decides not to use any leverage the final profit decreases 10 times. On the other hand, with using the leverage 1 to 20 the final profit doubles itself. Of course, the use of higher leverages is riskier as also dropdowns are higher. There is, for example, in Fig. 5 a dropdown circa 6% around 600 trades. With the use of leverage 1 to 20 this dropdown rises to 12%.

128000 combinations of model's parameters have been calculated. Figures 5-8 describe some interesting cases of the prediction behavior of the model with the transaction cost included on the EUR/USD currency rate of forex market. Figures 5,6 describe the model (one set of parameters) with the requirement that the fall down must not be higher than 5%. The best profit achieved in this case is 12%.

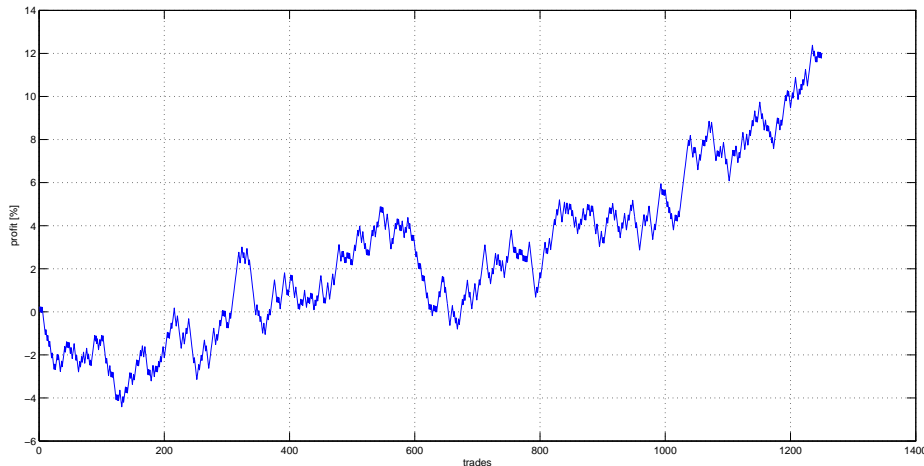


FIG. 5. The profit of the model on the EUR/USB currency rate with transaction costs included dependence on trades for one year period.

In order to sort out best combinations of parameters it is useful to use the statistical quantity called Sharpe ratio. The Sharpe ratio is a measure of the excess return per unit of risk in a trading strategy and is defined as

$$S = \frac{E(R - R_f)}{\sigma}, \quad (27)$$

where R is the asset return, R_f is the return on a benchmark asset (risk free), $E(R - R_f)$ is the mean value of the excess of the asset return over the benchmark return, and σ is the standard deviation of the excess of the asset return.

Figure 7 shows the case where the Sharpe ratio has the highest value from all sets of the calculated parameters. One year profit is around 26% and the maximum loss is slightly over 5%. Figure 8 describes the case with the requirements to high value of Sharpe ratio and with aim to gain profit over 50%.

There exist sufficiently enough cases with high Sharpe ratio which leads to enhancement of the model to create self-education model. This enhancement takes some ticks of data, finds out the best case of parameters (high Sharpe ratio and also high profit) and starts trading with these parameters for some period. Meanwhile, trading with previously found parameters model is looking for a new best combination of parameters. Figure 9 describes this self-education model where parameters are not chosen and the model itself finds the best one from the financial data and is subsequently looking for the best values for the next trading strategy.

V. CONCLUSIONS

The model of the strings allows one to manipulate with the information stored along several extra dimensions. We started from the theory of the 1-end-point and 2-end-point open string and continued with partially compactified strings that satisfy the Dirichlet and Neumann boundary conditions.

We have shown that the string theory may motivate the adoption of the nonlinear techniques of the data analysis with a minimum impact of justification parameters. The numerical study revealed interesting fundamental statistical properties of the maps from the data onto string-like objects. The main point here is that the string map gives a geometric interpretation of the information value of the data. The results led us to believe that our ideas and methodology can contribute to the solution of the problem of the robust portfolio selection. We believe that the method affords potential to be used in practical applications, where arbitrage selection bias should be taken into account.

We established two different string prediction models to predict the behavior of forex financial market. The first model is based on the correlation function as an invariant and the second one is an application based on the deviations from the closed string/pattern form. We found the difference between these two approaches. The first model cannot predict the behavior of the forex market with good efficiency in comparison with the second one which, moreover, is able to make relevant profit per year. From the results described we can conclude that the invariant model as one

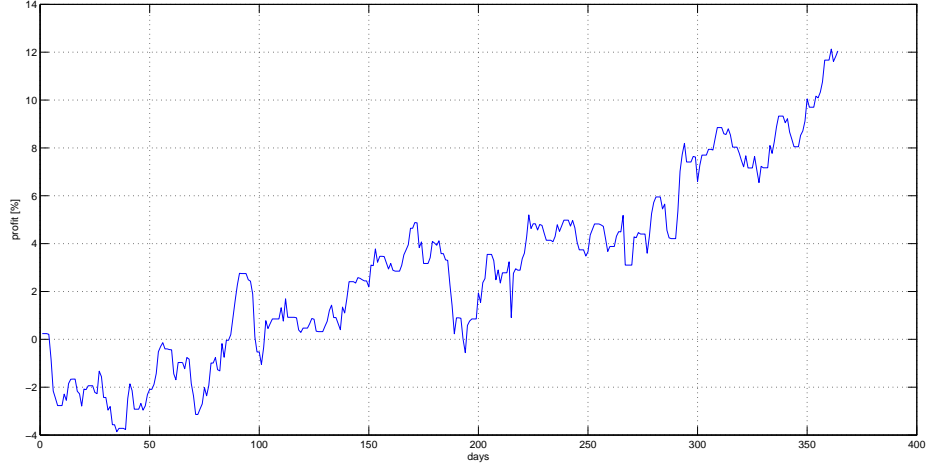


FIG. 6. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on days for one year period.

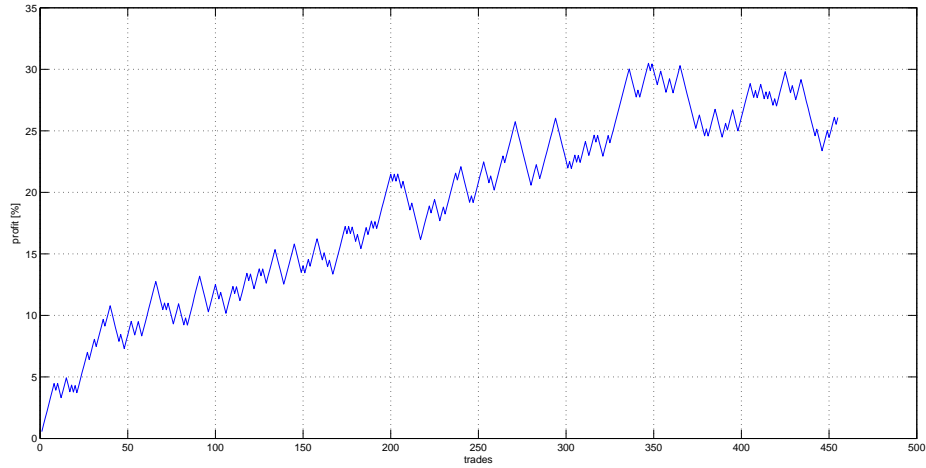


FIG. 7. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

step price prediction is not sufficient for big dynamic changes of the current prices on finance market. As can be seen in Figs. 3,4 when the transaction costs are switched off the model has some tendency to make a profit or at least preserve fortune. It means that it could be also useful but for the other kind of data, where the dynamics of changes are slower, e.g. for energetic [10] or seismographic data [11] with longer periods of changes. The profit per year from the second string model was obtained from approximately 15 % up to almost 100 % depending on the parameter set from the data we have chosen. This model is established very well on finance market and is useful to predict future prices for the trading strategy. Of course we need to test the model much more and with the flow of new financial data the model will be optimized better. Finally, we can conclude that with these models the string theory gains a real sense and can be applied not only in high energy physics but also in the finance market. Maybe this is some kind of proof that the string theory is a good candidate for the fundament of matter itself. The presented models are universal and could be also useful for predictions of other kind of stochastic data. The self-educated models presented in Fig. 9 are very useful because they are able to find the best parameter set from data, learn about the prices and utilize these information for the next trading strategy.

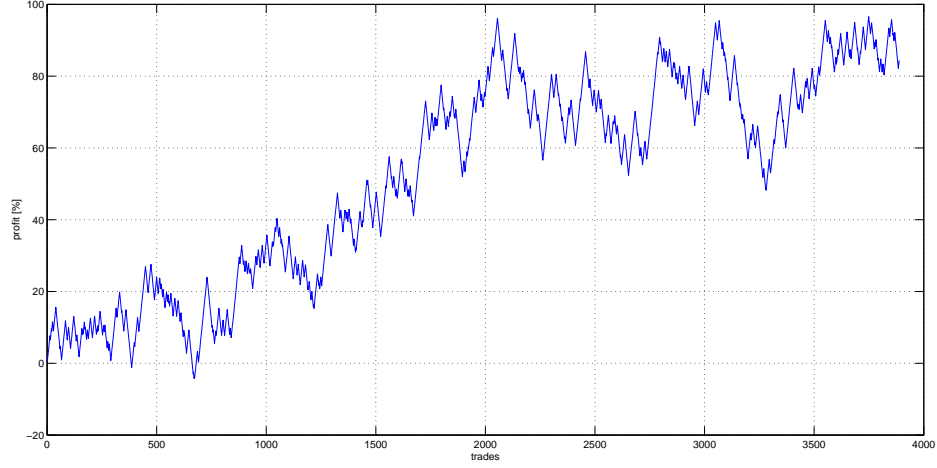


FIG. 8. The profit of the model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

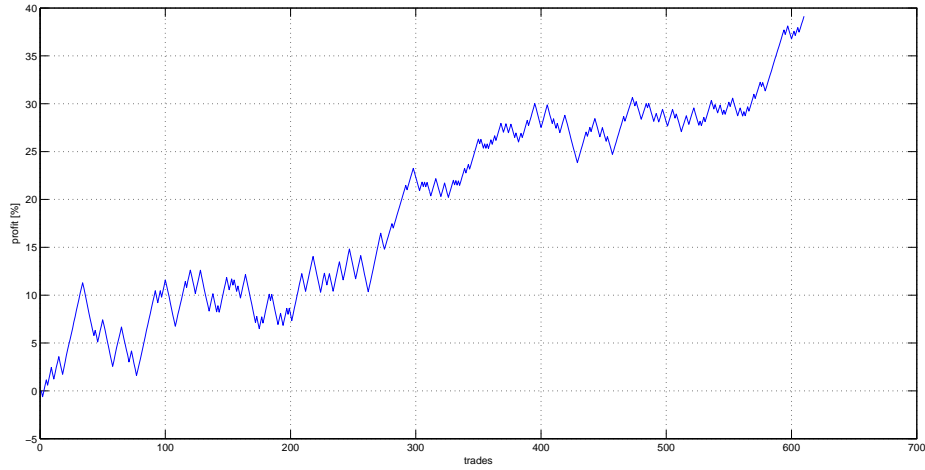


FIG. 9. The profit of the self education model on the EUR/USD currency rate with transaction costs included dependence on trades for one year period.

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